Week 05 – Loan

Loan exercise

A loan of **10,000** is repaid by **5 annual level payments** at an effective annual interest rate of **3%**.

Find:

1. The annual level payment.
2. The amortization table.
3. The outstanding loan balance after 2 payments, B_2, using:
   - prospective method
   - retrospective method
4. The interest portion of the 3rd payment.
5. The principal portion of the 3rd payment.
6. The outstanding loan balance at time 3 using

$$B_3 = B_2 + I_3 - R$$

7. The ratio of principal repaid in the 3rd payment to principal repaid in the 2nd payment, and show that it equals

$$1 + i$$

8. Check all of the above using the **BA II Plus calculator** with the **AMORT** function.
9. Find the **total principal repaid** and **total interest paid**, and compare:
   - calculator method
   - by-hand method without using the amortization table

Solution

1) Annual level payment

$$L = 10000, \quad i = 0.03, \quad n = 5$$

$$R = \frac{10000}{a_{\overline{5}|}}$$

with

$$a_{\overline{5}|} = \frac{1 - (1.03)^{-5}}{0.03} = 4.579707$$

Thus,

$$R = \frac{10000}{4.579707} = 2183.545714$$

$$\boxed{R \approx 2183.55}$$

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2) Amortization table

Payment time	Beginning balance	Interest	Payment	Principal repaid	Ending balance
0	10,000.00	-	-	-	10,000.00
1	10,000.00	300.00	2,183.55	1,883.55	8,116.45
2	8,116.45	243.49	2,183.55	1,940.05	6,176.40
3	6,176.40	185.29	2,183.55	1,998.25	4,178.15
4	4,178.15	125.34	2,183.55	2,058.20	2,119.95
5	2,119.95	63.60	2,183.55	2,119.95	0.00

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3) Outstanding balance after 2 payments, B_2

Prospective method

After 2 payments, 3 payments remain, so

$$B_2 = R\, a_{\overline{3}|}$$

$$a_{\overline{3}|} = \frac{1-(1.03)^{-3}}{0.03} = 2.828611$$

Therefore,

$$B_2 = 2183.545714(2.828611) = 6176.4022$$

$$\boxed{B_2 \approx 6176.40}$$

Retrospective method

$$B_2 = 10000(1.03)^2 - R\, s_{\overline{2}|}$$

where

$$s_{\overline{2}|} = \frac{(1.03)^2 - 1}{0.03} = 2.03$$

So,

$$B_2 = 10000(1.03)^2 - 2183.545714(2.03) = 6176.4022$$

$$\boxed{B_2 \approx 6176.40}$$

4) Interest portion of the 3rd payment

We already have

$$I_3 = iB_2 = 0.03(6176.4022) = 185.2921$$

$$\boxed{I_3 \approx 185.29}$$

Also, for a level payment loan,

$$I_t = R\left(1 - v^{n-t+1}\right)$$

So here,

$$I_3 = R(1 - v^3)$$

$$= 2183.545714\left(1 - (1.03)^{-3}\right)$$

$$= 2183.545714(1 - 0.915141659)$$

$$= 2183.545714(0.084858341)$$

$$= 185.2921$$

$$\boxed{I_3 \approx 185.29}$$

So both methods agree.

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5) Principal portion of the 3rd payment

We already have

$$P_3 = R - I_3$$

$$P_3 = 2183.545714 - 185.292066 = 1998.253648$$

$$\boxed{P_3 \approx 1998.25}$$

Also, for a level payment loan,

$$P_t = Rv^{n-t+1}$$

So here,

$$P_3 = Rv^3$$

$$= 2183.545714(1.03)^{-3}$$

$$= 2183.545714(0.915141659)$$

$$= 1998.2536$$

$$\boxed{P_3 \approx 1998.25}$$

Again, both methods agree.

6) Outstanding balance at time 3

Using

$$B_3 = B_2 + I_3 - R$$

we get

$$B_3 = 6176.4022 + 185.2921 - 2183.5457 = 4178.1486$$

$$\boxed{B_3 \approx 4178.15}$$

This is the same as

$$B_3 = B_2 - P_3$$

7) Ratio of principal repaid in the 3rd payment to principal repaid in the 2nd payment

First find the principal repaid in the 2nd payment.

At time 1, the balance is

$$B_1 = 10000(1.03) - 2183.545714 = 8116.454286$$

So

$$I_2 = 0.03(8116.454286) = 243.493629$$

and

$$P_2 = R - I_2 = 2183.545714 - 243.493629 = 1940.052085$$

Now,

$$\frac{P_3}{P_2} = \frac{1998.253648}{1940.052085} = 1.03$$

Thus,

$$\boxed{\frac{P_3}{P_2} = 1.03 = 1 + i}$$

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8) BA II Plus calculator check

Step A: Enter TVM first

BA II Plus requires solving the TVM problem before using AMORT.

Enter:

- $N = 5$
- $I/Y = 3$
- $PV = 10000$
- $FV = 0$
- $PMT = ?$

Because this is a loan, the calculator may return PMT as negative if PV is positive. That sign issue is normal.

You should get

$$PMT = -2183.545714$$

So the payment amount is

$$\boxed{2183.55}$$

Step B: Use AMORT

Go to

$$\text{2ND} \rightarrow \text{AMORT}$$

Definitions:

- **P1** = starting period
- **P2** = ending period
- **BAL** = balance remaining after P2
- **PRN** = principal repaid from P1 to P2
- **INT** = interest paid from P1 to P2

Check the 1st payment

Set:

- $P1 = 1$
- $P2 = 1$

Then scroll:

- BAL $= 8116.45$
- PRN $= 1883.55$
- INT $= 300.00$

So after 1 payment,

$$B_1 = 8116.45$$

Check the 2nd payment

Set:

- $P1 = 2$
- $P2 = 2$

Then:

- BAL $= 6176.40$
- PRN $= 1940.05$
- INT $= 243.49$

So

$$\boxed{B_2 = 6176.40}$$

and

$$\boxed{P_2 = 1940.05, \quad I_2 = 243.49}$$

Check the 3rd payment

Set:

- $P1 = 3$
- $P2 = 3$

Then:

- BAL $= 4178.15$
- PRN $= 1998.25$
- INT $= 185.29$

So

$$\boxed{B_3 = 4178.15, \quad P_3 = 1998.25, \quad I_3 = 185.29}$$

This confirms parts 4, 5, and 6.

Check total principal and total interest

Set:

- $P1 = 1$
- $P2 = 5$

Then the AMORT worksheet gives totals over all payments:

- PRN $= 10000$
- INT $= 917.73$

So

$$\boxed{\text{Total principal repaid} = 10000}$$

$$\boxed{\text{Total interest paid} = 917.73}$$

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9) Total principal repaid and total interest paid

Compare calculator and by hand

Calculator method

From AMORT with $P1 = 1, P2 = 5$:

$$\boxed{\text{Total principal repaid} = 10000}$$

$$\boxed{\text{Total interest paid} = 917.73}$$

By hand method without amortization table

Total principal repaid

For a fully repaid loan, the total principal repaid must equal the original loan amount:

$$\boxed{\text{Total principal repaid} = 10000}$$

Total interest paid

Total amount paid over 5 years is

$$5R = 5(2183.545714) = 10917.72857$$

Since

$$\text{Total paid} = \text{Principal} + \text{Interest}$$

we have

$$\text{Total interest} = 10917.72857 - 10000 = 917.72857$$

Thus,

$$\boxed{\text{Total interest paid} \approx 917.73}$$

Comparison summary

Quantity	By hand	BA II Plus
Level payment R	2183.55	2183.55
B_2	6176.40	6176.40
I_3	185.29	185.29
P_3	1998.25	1998.25
B_3	4178.15	4178.15
Total principal repaid	10000.00	10000.00
Total interest paid	917.73	917.73

Nice takeaway formulas

For a level payment loan:

$$R = \frac{L}{a_{\overline{n}|}}$$

$$B_t = R\, a_{\overline{n-t}|} \quad \text{(prospective)}$$

$$B_t = L(1+i)^t - R\, s_{\overline{t}|} \quad \text{(retrospective)}$$

$$I_t = iB_{t-1}$$

$$P_t = R - I_t$$

$$B_t = B_{t-1} + I_t - R$$

$$\frac{P_{t+1}}{P_t} = 1 + i$$

$$\text{Total interest} = nR - L$$

19.
Project P requires an investment of 4000 today. The investment pays 2000 one year from today and 4000 two years from today.

Project Q requires an investment of X two years from today. The investment pays 2000 today and 4000 one year from today.

The net present values of the two projects are equal at an annual effective interest rate of 10%.

Calculate X.

19. Solution: D

Equate net present values:

$$-4000 + 2000v + 4000v^2 = 2000 + 4000v - Xv^2$$

$$\frac{4000 + X}{1.21} = 6000 + \frac{2000}{1.1}$$

$$X = 5460.$$

27.

An investor pays 100,000 today for a 4-year investment that returns cash flows of 60,000 at the end of each of years 3 and 4. The cash flows can be reinvested at 4.0% per annum effective.

Using an annual effective interest rate of 5.0%, calculate the net present value of this investment today.

27. Solution: C.

The first cash flow of 60,000 at time 3 earns 2400 in interest for a time 4 receipt of 62,400. Combined with the final payment, the investment returns 122,400 at time 4. The present value is $122,400(1.05)^{-4} = 100,699$. The net present value is 699.

121.
A company is considering investing in a particular project. The project requires an investment of X today. Additional investments are required at the beginning of each of the next five years, with each year's investment 5% greater than the previous year's investment.

The investment is expected to produce an income of 100 per year at the end of each year forever, with the first payment expected at the end of the first year.

At an annual effective interest rate of 10.25%, the project has a net present value of zero.

Calculate X.

121. Solution: A
The present value of the income is $100a_{\overline{\infty}|0.1025} = 100/0.1025 = 975.61$. The present value of the investment is

$$X\left[1+1.05/1.1025+(1.05/1.1025)^2+(1.05/1.1025)^3+(1.05/1.1025)^4+(1.05/1.1025)^5\right]$$

$$= X[1+1.05^{-1}+1.05^{-2}+1.05^{-3}+1.05^{-4}+1.05^{-5}] = X\frac{1-1.05^{-6}}{1-1.05^{-1}} = 5.3295X.$$

Then 975.61=5.3295X for X = 183.06.

241.
An investor pays 4000 today for a three-year investment that returns cash flows of 1400 at the end of each year. The cash flows can be reinvested at the positive annual effective interest rate of i. Using an annual effective rate of interest of 4%, the net present value of this investment is 0.

Calculate i.

241. Solution: E

Solution: $4000(1.04^3) = 1400[1 + (1+i) + (1+i)^2]$, or i solves the quadratic equation: $i^2 + 3i - 0.2139 = 0$. Thus, $i = 6.97\%$ because the other root is negative.

180.

A company is considering a project that will require an initial investment of 600 and additional investments of 100 and 50 at the end of years one and two, respectively. It is expected that revenue from this project will be 150 per year for five years, beginning one year from the initial investment.

Assuming an annual effective rate of 15%, calculate the net present value of this project.

180. Solution: A

$$NPV = -600 + \frac{150-100}{1.15} + \frac{150-50}{1.15^2} + \frac{150}{1.15^3} + \frac{150}{1.15^4} + \frac{150}{1.15^5} = -221.94$$

6.
A 20-year loan of 1000 is repaid with payments at the end of each year.

Each of the first ten payments equals 150% of the amount of interest due. Each of the last ten payments is X.

The lender charges interest at an annual effective rate of 10%.

Calculate X.

6. Solution: D

For the first 10 years, each payment equals 150% of interest due. The lender charges 10%, therefore 5% of the principal outstanding will be used to reduce the principal.

At the end of 10 years, the amount outstanding is $1000(1-0.05)^{10} = 598.74$.

Thus, the equation of value for the last 10 years using a comparison date of the end of year 10 is

$598.74 = Xa_{\overline{10}|10\%} = 6.1446X$

$X = 97.44$.

12.
A 10-year loan of 2000 is to be repaid with payments at the end of each year. It can be repaid under the following two options:

(i) Equal annual payments at an annual effective interest rate of 8.07%.

(ii) Installments of 200 each year plus interest on the unpaid balance at an annual effective interest rate of i.

The sum of the payments under option (i) equals the sum of the payments under option (ii).

Calculate i.

12. Solution: B

Option 1: $2000 = Pa_{\overline{10}|0.0807}$

$P = 299 \Rightarrow$ Total payments $= 2990$

Option 2: Interest needs to be $2990 - 2000 = 990$

$990 = i[2000 + 1800 + 1600 + \cdots + 200]$

$= 11{,}000i$

$i = 0.09 = 9.00\%$

13.
A loan is amortized over five years with monthly payments at an annual nominal interest rate of 9% compounded monthly. The first payment is 1000 and is to be paid one month from the date of the loan. Each succeeding monthly payment will be 2% lower than the prior payment.

Calculate the outstanding loan balance immediately after the 40th payment is made.

13. Solution: B
Monthly payment at time t is $1000(0.98)^{t-1}$.
Because the loan amount is unknown, the outstanding balance must be calculated prospectively. The value at time 40 months is the present value of payments from time 41 to time 60:

$$OB_{40} = 1000[0.98^{40}v^{1} + \cdots + 0.98^{59}v^{20}]$$

$$= 1000\frac{0.98^{40}v^{1} - 0.98^{60}v^{21}}{1-0.98v},\ v = 1/(1.0075)$$

$$= 1000\frac{0.44238 - 0.25434}{1-0.97270} = 6888.$$

21.
Seth, Janice, and Lori each borrow 5000 for five years at an annual nominal interest rate of 12%, compounded semi-annually.

Seth has interest accumulated over the five years and pays all the interest and principal in a lump sum at the end of five years.

Janice pays interest at the end of every six-month period as it accrues and the principal at the end of five years.

Lori repays her loan with 10 level payments at the end of every six-month period.

Calculate the total amount of interest paid on all three loans.

21. Solution: D

The given information yields the following amounts of interest paid:

$$\text{Seth} = 5000\left(\left(1+\frac{0.12}{2}\right)^{10}-1\right) = 8954.24 - 5000 = 3954.24$$

$$\text{Janice} = 5000(0.06)(10) = 3000.00$$

$$\text{Lori} = P(10) - 5000 = 1793.40 \text{ where } P = \frac{5000}{a_{\overline{10}|6\%}} = 679.35$$

The sum is 8747.64.

23.
Ron is repaying a loan with payments of 1 at the end of each year for n years. The annual effective interest rate on the loan is i. The amount of interest paid in year t plus the amount of principal repaid in year $t+1$ equals X.

Determine which of the following is equal to X.

(A) $1+\dfrac{v^{n-t}}{i}$

(B) $1+\dfrac{v^{n-t}}{d}$

(C) $1+v^{n-t}i$

(D) $1+v^{n-t}d$

(E) $1+v^{n-t}$

23. Solution: D

Year t interest is $ia_{\overline{n-t+1}|i} = 1 - v^{n-t+1}$.

Year t+1 principal repaid is $1-(1-v^{n-t}) = v^{n-t}$.

$X = 1 - v^{n-t+1} + v^{n-t} = 1 + v^{n-t}(1-v) = 1 + v^{n-t}d.$

33.
Seth borrows X for four years at an annual effective interest rate of 8%, to be repaid with equal payments at the end of each year. The outstanding loan balance at the end of the third year is 559.12.

Calculate the principal repaid in the first payment.

33. Solution: A

The outstanding balance is the present value of future payments. With only one future payment, that payment must be 559.12(1.08) = 603.85. The amount borrowed is $603.85a_{\overline{4}|0.08} = 2000$. The first payment has 2000(0.08) = 160 in interest, thus the principal repaid is 603.85 – 160 = 443.85.

Alternatively, observe that the principal repaid in the final payment is the outstanding loan balance at the previous payment, or 559.12. Principal repayments form a geometrically decreasing sequence, so the principal repaid in the first payment is $559.12/1.08^3 = 443.85$.

48.

Tanner takes out a loan today and repays the loan with eight level annual payments, with the first payment one year from today. The payments are calculated based on an annual effective interest rate of 4.75%. The principal portion of the fifth payment is 699.68.

Calculate the total amount of interest paid on this loan.

48. **Solution: A**

$$699.68 = Pv^{8-5+1}$$

$$P = 842.39 \text{ (annual payment)}$$

$$P_1 = \frac{699.68}{1.0475^4} = 581.14$$

$$I_1 = 842.39 - 581.14 = 261.25$$

$$L = \frac{261.25}{0.0475} = 5500 \text{ (loan amount)}$$

$$\text{Total interest} = 842.39(8) - 5500 = 1239.12$$

60.
A borrower takes out a 15-year loan for 400,000, with level end-of-month payments, at an annual nominal interest rate of 9% convertible monthly.

Immediately after the 36th payment, the borrower decides to refinance the loan at an annual nominal interest rate of j, convertible monthly. The remaining term of the loan is kept at twelve years, and level payments continue to be made at the end of the month. However, each payment is now 409.88 lower than each payment from the original loan.

Calculate j.

60. Solution: D

The initial level monthly payment is

$$R = \frac{400,000}{a_{\overline{15\times 12}|0.09/12}} = \frac{400,000}{a_{\overline{180}|0.0075}} = 4,057.07.$$

The outstanding loan balance after the 36th payment is

$$B_{36} = Ra_{\overline{180-36}|0.0075} = 4,057.07a_{\overline{144}|0.0075} = 4,057.07(87.8711) = 356,499.17.$$

The revised payment is $4,057.07 - 409.88 = 3,647.19$.

Thus,

$$356,499.17 = 3,647.19a_{\overline{144}|j/12}$$

$$a_{\overline{144}|j/12} = 356,499.17 / 3,647.19 = 97.7463.$$

Using the financial calculator, $j/12 = 0.575\%$, for $j = 6.9\%$.

64.

A borrower takes out a 50-year loan, to be repaid with payments at the end of each year. The loan payment is 2500 for each of the first 26 years. Thereafter, the payments decrease by 100 per year. Interest on the loan is charged at an annual effective rate of i ($0\% < i < 10\%$).
The principal repaid in year 26 is X.

Determine the amount of interest paid in the first year.

(A) Xv^{25}
(B) $2500v^{25} - Xv^{25}$
(C) $2500 - X$
(D) $2500 - Xv^{25}$
(E) $25Xi$

64. Solution: D

The outstanding balance at time 25 is $100(Da)_{\overline{25|}} = 100\dfrac{25 - a_{\overline{25|}}}{i}$. The principle repaid in the 26th payment is $X = 2500 - i(100)\dfrac{25 - a_{\overline{25|}}}{i} = 2500 - 2500 + 100a_{\overline{25|}} = 100a_{\overline{25|}}$. The amount borrowed is the present value of all 50 payments, $2500a_{\overline{25|}} + v^{25}100(Da)_{\overline{25|}}$. Interest paid in the first payment is then

$$\begin{aligned}
&i\left[2500a_{\overline{25|}} + v^{25}100(Da)_{\overline{25|}}\right] \\
&= 2500(1 - v^{25}) + 100v^{25}(25 - a_{\overline{25|}}) \\
&= 2500 - 2500v^{25} + 2500v^{25} - v^{25}100a_{\overline{25|}} \\
&= 2500 - Xv^{25}.
\end{aligned}$$

67.
A loan of 10,000 is repaid with a payment made at the end of each year for 20 years. The payments are 100, 200, 300, 400, and 500 in years 1 through 5, respectively. In the subsequent 15 years, equal annual payments of X are made.
The annual effective interest rate is 5%.

67. Solution: E

$$10{,}000 = 100(Ia)_{\overline{5}|} + Xv^5 a_{\overline{15}|} = 100\left(\frac{\ddot{a}_{\overline{5}|} - 5v^5}{0.05}\right) + Xv^5 a_{\overline{15}|}$$

$$10{,}000 = 1256.64 + 8.13273X$$

$$1075 = X$$

69.

A borrower takes out a 15-year loan for 65,000, with level end-of-month payments. The annual nominal interest rate of the loan is 8%, convertible monthly.

Immediately after the 12th payment is made, the remaining loan balance is reamortized. The maturity date of the loan remains unchanged, but the annual nominal interest rate of the loan is changed to 6%, convertible monthly.

Calculate the new end-of-month payment.

69. Solution: E

The monthly payment on the original loan is $\dfrac{65{,}000}{a_{\overline{180}|8/12\%}} = 621.17$. After 12 payments the outstanding balance is $621.17a_{\overline{168}|8/12\%} = 62{,}661.40$. The revised payment is $\dfrac{62{,}661.40}{a_{\overline{168}|6/12\%}} = 552.19$.

82.
A 30-year annuity is arranged to pay off a loan taken out today at a 5% annual effective interest rate. The first payment of the annuity is due in ten years in the amount of 1,000. The subsequent payments increase by 500 each year.

Calculate the amount of the loan.

82. Solution: D
The amount of the loan is the present value of the deferred increasing annuity:

$$(1.05)^{-10}\left[500\ddot{a}_{\overline{30}|0.05}+500(I\ddot{a})_{\overline{30}|0.05}\right]=(1.05^{-10})(500)\left[\ddot{a}_{\overline{30}|0.05}+\frac{\ddot{a}_{\overline{30}|0.05}-30(1.05)^{-30}}{0.05/1.05}\right]=64,257.$$

87.
A company takes out a loan of 15,000,000 at an annual effective discount rate of 5.5%. You are given:

i) The loan is to be repaid with n annual payments of 1,200,000 plus a drop payment one year after the nth payment.

ii) The first payment is due three years after the loan is taken out.

Calculate the amount of the drop payment.

87. Solution: D

The effective annual interest rate is $i = (1-d)^{-1} - 1 = (1-0.055)^{-1} - 1 = 5.82\%$

The balance on the loan at time 2 is $15,000,000(1.0582)^2 = 16,796,809$.

The number of payments is given by $1,200,000 a_{\overline{n|}} = 16,796,809$ which gives $n = 29.795$ => 29 payments of 1,200,000. The final equation of value is

$1,200,000 a_{\overline{29|}} + X(1.0582)^{-30} = 16,796,809$

$X = (16,796,809 - 16,621,012)(5.45799) = 959,490.$

88.
Tim takes out an n-year loan with equal annual payments at the end of each year.

The interest portion of the payment at time $(n - 1)$ is equal to 0.5250 of the interest portion of the payment at time $(n - 3)$ and is also equal to 0.1427 of the interest portion of the first payment.

Calculate n.

88. **Solution: C**

$$1-v^2 = 0.525(1-v^4) \Rightarrow 1 = 0.525(1+v^2) \Rightarrow v^2 = 0.90476 \Rightarrow v = 0.95119$$

$$1-v^2 = 0.1427(1-v^n) \Rightarrow 1-v^n = (1-0.90476)/0.1427 = 0.667414 \Rightarrow v^n = 0.332596$$

$$n = \ln(0.332596)/\ln(0.95119) = 22$$

89. Solution: C

The monthly payment is $200,000/a_{\overline{360}|0.005}=1199.10$. Using the equivalent annual effective rate of 6.17%, the present value (at time 0) of the five extra payments is 41,929.54 which reduces the original loan amount to 200,000 – 41,929.54 = 158,070.46. The number of months required is the solution to $158,070.46=1199.10a_{\overline{n}|0.005}$. Using calculator, $n = 215.78$ months are needed to pay off this amount. So there are 215 full payments plus one fractional payment at the end of the 216th month, which is December 31, 2020.

89.

On January 1, 2003 Mike took out a 30-year mortgage loan in the amount of 200,000 at an annual nominal interest rate of 6% compounded monthly. The loan was to be repaid by level end-of-month payments with the first payment on January 31, 2003.

Mike repaid an extra 10,000 in addition to the regular monthly payment on each December 31 in the years 2003 through 2007.

Determine the date on which Mike will make his last payment (which is a drop payment).

(A) July 31, 2013
(B) November 30, 2020
(C) December 31, 2020
(D) December 31, 2021
(E) January 31, 2022

90.
A 5-year loan of 500,000 with an annual effective discount rate of 8% is to be repaid by level end-of-year payments.

If the first four payments had been rounded up to the next multiple of 1,000, the final payment would be X.

Calculate X.

90. Solution: D

The annual effective interest rate is 0.08/(1 – 0.08) = 0.08696. The level payments are $500,000 / a_{\overline{5}|0.08696} = 500,000 / 3.9205 = 127,535$. This rounds up to 128,000. The equation of value for X is

$128,000 a_{\overline{4}|0.08696} + X(1.08696)^{-5} = 500,000$

$X = (500,000 - 417,466.36)(1.51729) = 125,227.$

92.

A loan of X is repaid with level annual payments at the end of each year for 10 years. You are given:

i) The interest paid in the first year is 3600; and

ii) The principal repaid in the 6^{th} year is 4871.

Calculate X.

92. Solution: D

Let P be the annual payment. The fifth line is obtained by solving a quadratic equation.

$$P(1-v^{10}) = 3600$$

$$Pv^{10-6+1} = 4871$$

$$\frac{1-v^{10}}{v^5} = \frac{3600}{4871}$$

$$1-v^{10} = 0.739068v^5$$

$$v^5 = 0.69656$$

$$v^{10} = 0.485195$$

$$i = 0.485195^{-1/10} - 1 = 0.075$$

$$X = P\frac{1-v^{10}}{i} = \frac{3600}{0.075} = 48,000$$

94.
Jeff has 8000 and would like to purchase a 10,000 bond. In doing so, Jeff takes out a 10 year loan of 2000 from a bank and will make interest-only payments at the end of each month at a nominal rate of 8.0% convertible monthly. He immediately pays 10,000 for a 10-year bond with a par value of 10,000 and 9.0% coupons paid monthly.

Calculate the annual effective yield rate that Jeff will realize on his 8000 over the 10-year period.

94. Solution: B
Jeff's monthly cash flows are coupons of 10,000(0.09)/12 = 75 less loan payments of 2000(0.08)/12 = 13.33 for a net income of 61.67. At the end of the ten years (in addition to the 61.67) he receives 10,000 for the bond less a 2,000 loan repayment. The equation is

$$8000 = 61.67 a_{\overline{120}|i^{(12)}/12} + 8000(1 + i^{(12)}/12)^{-120}$$

$$i^{(12)}/12 = 0.00770875$$

$$i = 1.00770875^{12} - 1 = 0.0965 = 9.65\%.$$